Control

27/10/2015 الثلاثاء د. عرفة محاضرة 4

Bode diagram :-

Stability

① Absolute Stability
- Routh.

② Relative stability
- Bode Diagram.
- polar plot..
- Nyquist.

كلما زادت ال gain margin وال phase margin
كلما زادت درجة استقرار النظام

in relative stability you know to what range or to what degree the system is stable throgh stability indicators (phase margin (PM) and gain margin (gm))

* Frequency Analysis

given T.F. in s-domain and we want to describe it in freq. domain

we replace $s \rightarrow j\omega$

given O.L.T.F $GH(s)$ to get Bode Diagram

① $s \rightarrow j\omega \Rightarrow GH(j\omega) = GH(s)\Big|_{s=j\omega}$

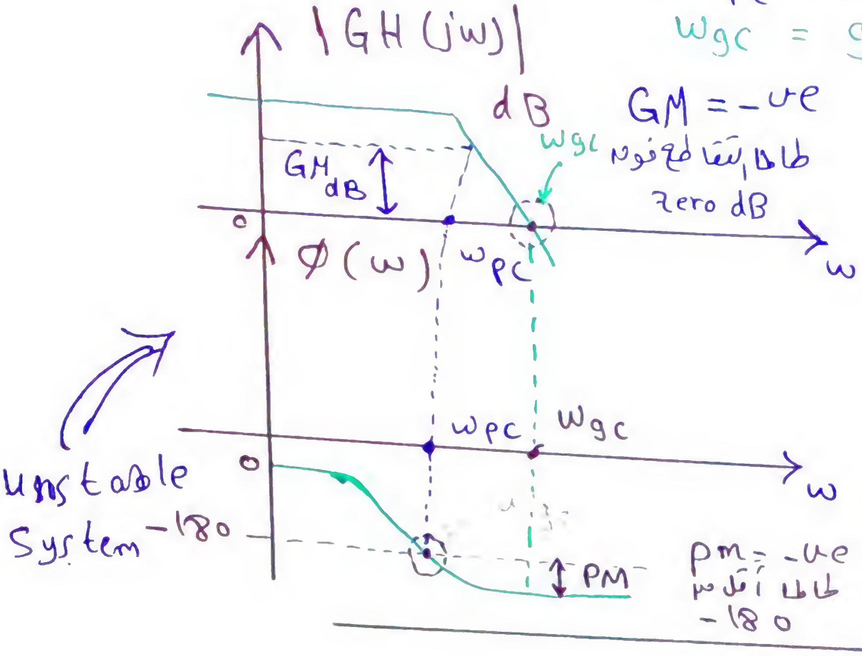
معادلة ال magnitude   معادلة ال phase

② $|GH(j\omega)|$, $\angle GH(j\omega)$

③ $|GH(j\omega)|_{dB} = 20 \log |GH(j\omega)|$

$\Longrightarrow$ Turn over

④ Diagram



w_{pc} = phase cross freq.

w_{gc} = gain cross freq.

بمعلومية ال magnitude

وال phase تقدر

تحدد ال Stability

كاملة بإزاي

$$| \ | = \sqrt{Real^2 + img^2}$$

$$\phi = \tan^{-1}\left(\frac{img}{Real}\right)$$

$$| \ |_{dB} = 0$$

$$\angle GH(jw) = -180$$

طالما أدرس ال system هدرس عن :

Ch. eq $\Rightarrow$ $1 + GH(s) = 0$

$$GH(s) = -1 + j0$$

Stability:-

① GM & PM > 0 (+ve) $\Rightarrow$ Stable

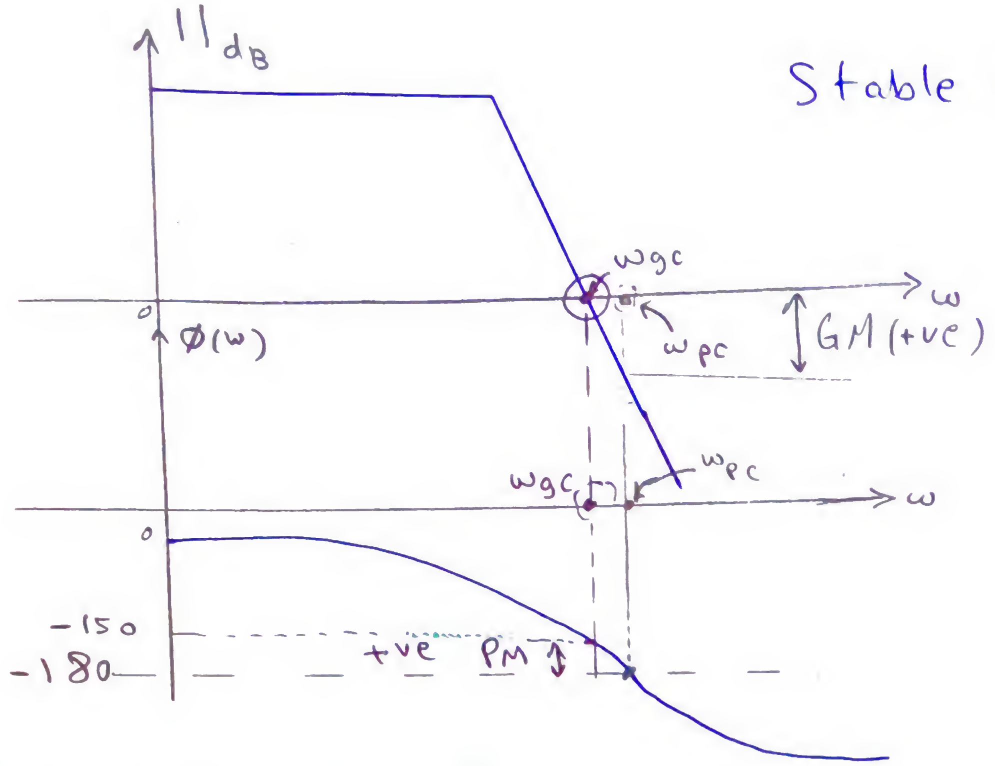
② GM & PM < 0 (-ve) $\Rightarrow$ unstable

③ GM_{dB} & PM = 0 $\Rightarrow$ critically stable

كلما زادت GM & PM زادت ال Stability

⟹ Turn over

Stable System

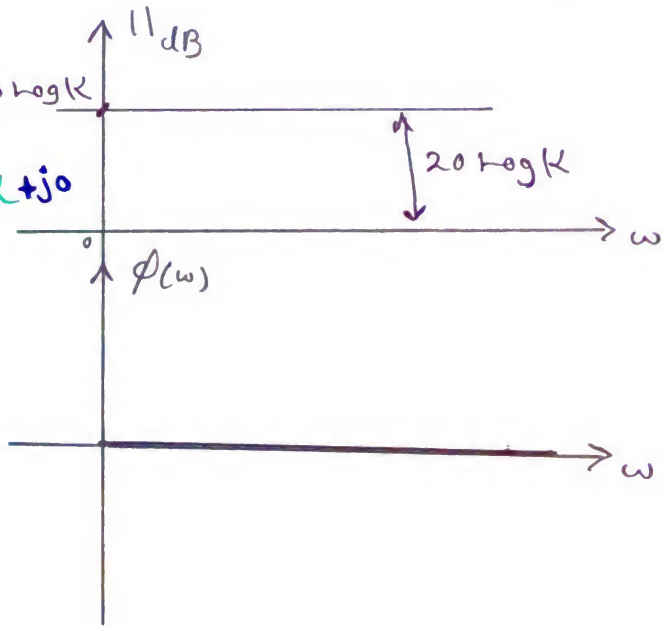


$$PM = 180 + \phi(\omega)\Big|_{\omega=\omega_{gc}}$$

* The open Loop T.F. $GH(S)$ Can be in the following forms:

① $GH(s) = K$

1. $s \longrightarrow j\omega \Rightarrow GH(j\omega) = K + j0$
2. $|GH(j\omega)| = K$
3. $|GH(j\omega)|_{dB} = 20 \log K$
4. $\phi(\omega) = \tan^{-1}\left(\frac{0}{K}\right) = 0$



② $GH(S) = \frac{1}{S}$

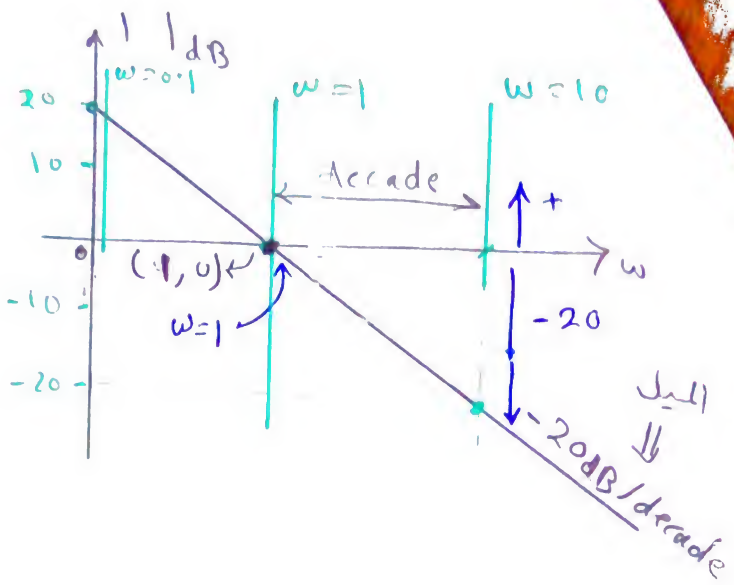
① $S \longrightarrow j\omega \Rightarrow GH(j\omega) = \frac{1}{j\omega}$

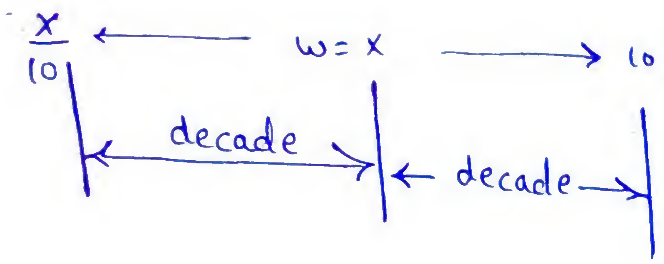
② $|GH(j\omega)| = \frac{1}{\omega}$

③ $|GH(j\omega)|_{dB} = 20 \operatorname{Log}\left(\frac{1}{\omega}\right)$

$= 20 \operatorname{Log} \omega^{(-1)}$

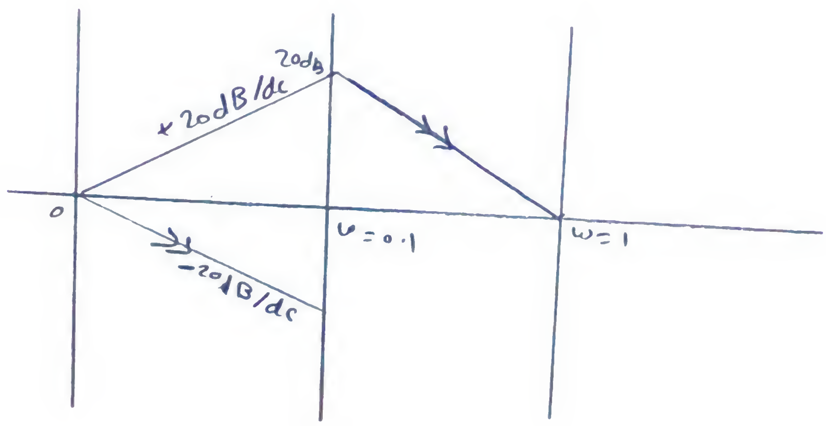
$= -20 \operatorname{Log} \omega$





ω	0.1	1	10	100
$\|\ \|_{dB}$	20dB	0	-20dB	-40dB

طريقة أخرى لرسم الميل: موازي



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③ $GH(s) = \frac{1}{s^2}$

① $s \to j\omega \Longrightarrow GH(j\omega) = \frac{1}{(j\omega)^2}$

$= \frac{1}{j\omega \cdot j\omega}$

② $|GH(j\omega)| = \frac{1}{\omega^2}$

③ $|\;|_{dB} = 20 \log\left(\frac{1}{\omega^2}\right)$

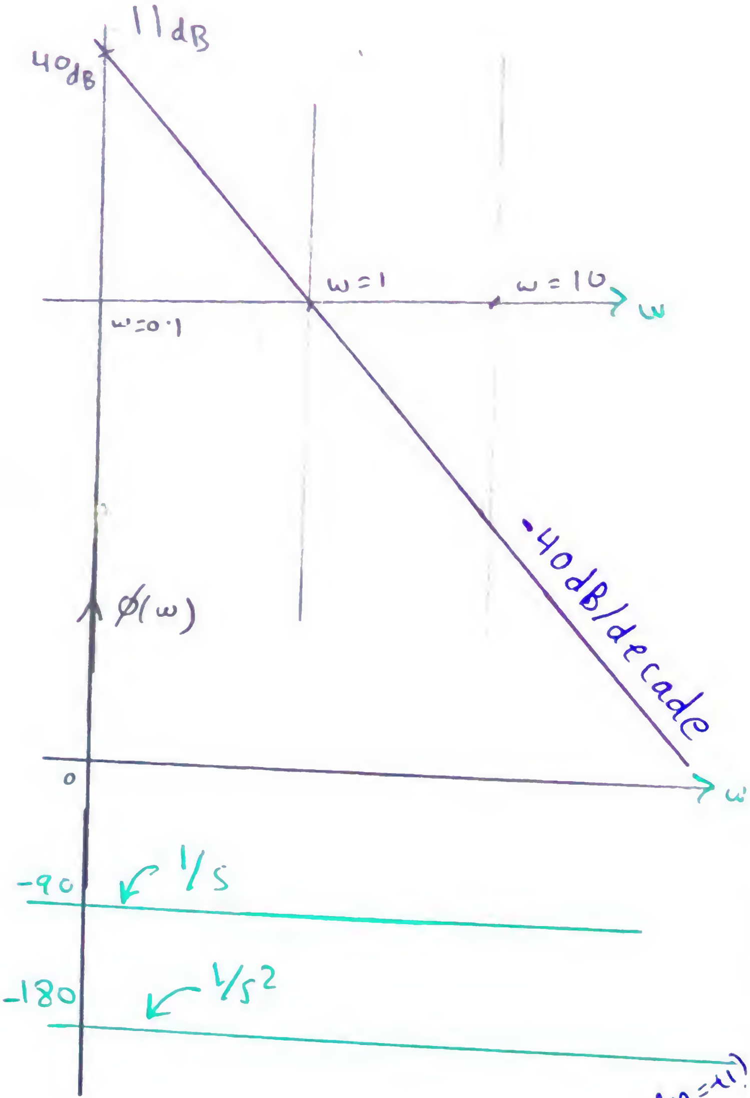
$= 20 \log \omega^{-2}$

$= -40 \log \omega$

يمكن الرسم بنفس الطريقتين السابقتين في المثال السابق

$\frac{1}{s} \Rightarrow \phi(\omega) = -90^\circ$

$\frac{1}{s^2} \Rightarrow \phi(\omega) = -180^\circ$



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④ $GH(s) = (s)^{\pm n}$

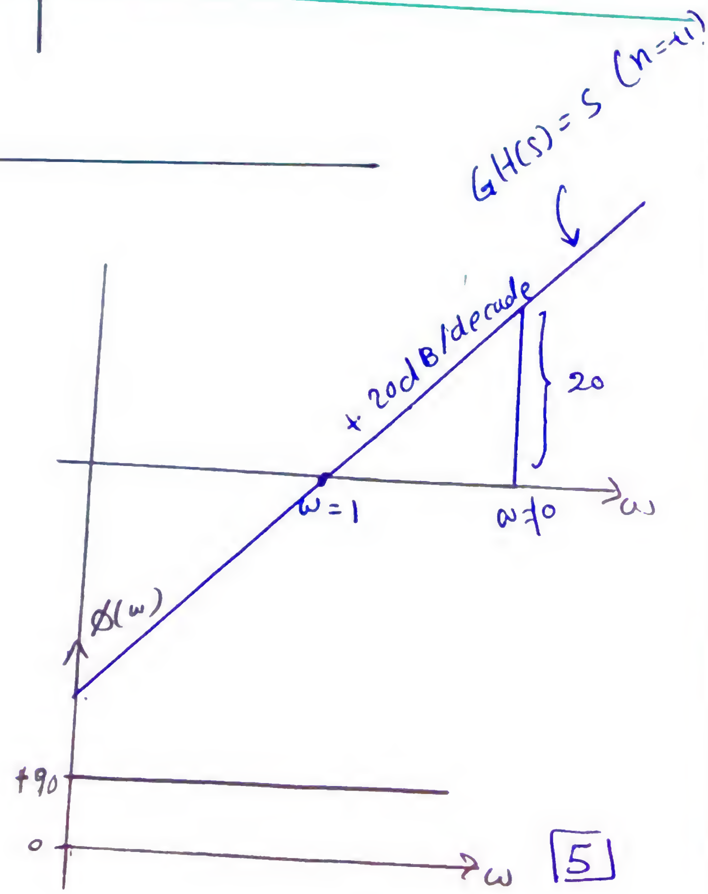
$|\;|_{dB} \Rightarrow$ خط مستقيم يمر ب $\omega = 1$

وميله $\pm 20n\ dB/decade$

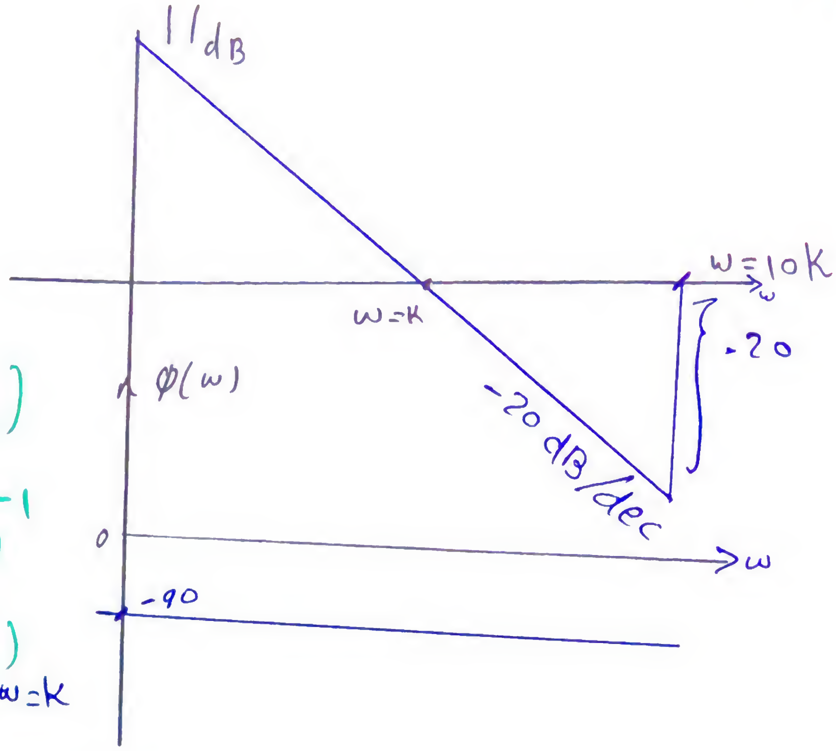
البسط ($+$)، المقام ($-$)

$\phi(\omega) = \pm 90n$

$\Longrightarrow$ turn over



⑤ $GH(s) = \dfrac{K}{s}$



- $s \to j\omega \Rightarrow GH(j\omega) = \dfrac{K}{j\omega}$

$$|GH(j\omega)| = \frac{K}{\omega}$$

$$|GH(j\omega)|_{dB} = 20\log\left(\frac{K}{\omega}\right)$$

$$= 20\log\left(\frac{\omega}{K}\right)^{-1}$$

$$= -20\log\left(\frac{\omega}{K}\right)$$

الميل

@ $\omega = K$

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⑥ $GH(s) = \dfrac{K}{s^2}$

خط مستقيم يمر ب $\omega = \sqrt{K}$ والميل $-40\,dB/decade$

وزاويتك $\phi(\omega)\ -180$

⑦ $GH(s) = \dfrac{K}{s^3}$

خط مستقيم يمر ب $\omega = \sqrt[3]{K}$ والميل $-60\,dB/decade$

وزاويته $\phi(\omega) = -270$

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⑧ $GH(s) = \left(1 + \dfrac{s}{c}\right)$

- $s \to j\omega$

$\Rightarrow GH(j\omega) = \left(1 + j\dfrac{\omega}{c}\right)$

- $|GH(j\omega)| = \sqrt{1 + \left(\dfrac{\omega}{c}\right)^2}$

- $\phi(\omega) = \tan^{-1}\left(\dfrac{\omega}{c}\right)$

$$|GH(j\omega)|_{dB} = 20\log\sqrt{1 + \left(\frac{\omega}{c}\right)^2}$$

نحللها على جزئين

مرة عند $\omega > c$

ومرة عند $\omega < c$

approximation

① $\omega < c \Rightarrow \left(\frac{\omega}{c}\right)^2 \ll 1$

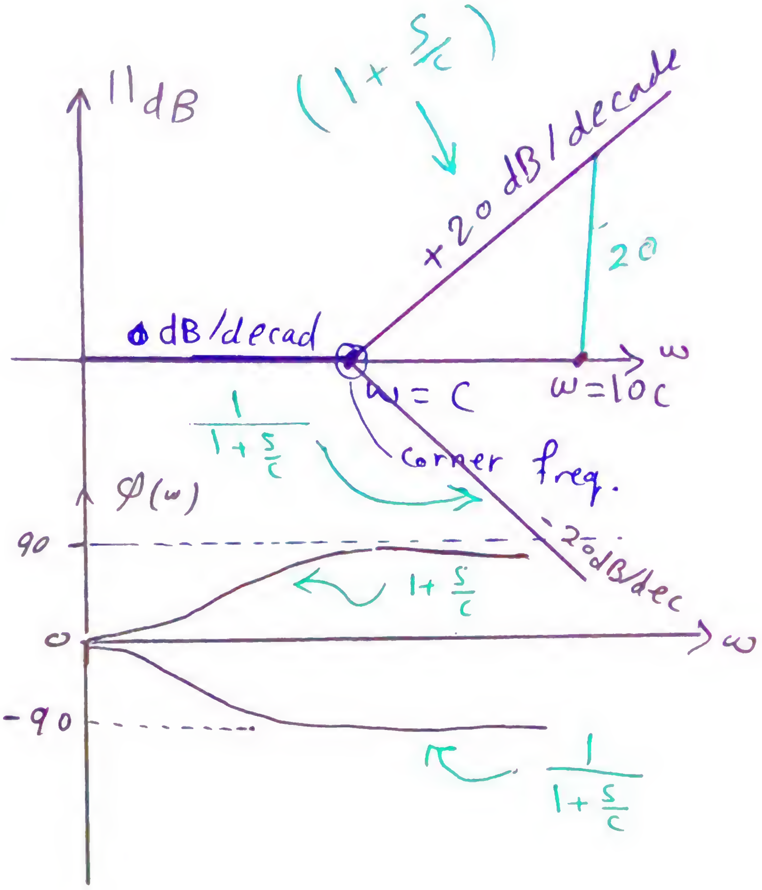
$$|GH(j\omega)|_{dB} = 20 \log \sqrt{1+0}$$
$$= 0\, dB$$

② $\omega > c \Rightarrow \left(\frac{\omega}{c}\right)^2 \gg 1$

$$|GH(j\omega)|_{dB} = 20 \log \sqrt{0 + \left(\frac{\omega}{c}\right)^2}$$
$$= 20 \log \left(\frac{\omega}{c}\right)$$

$\phi(\omega) \Rightarrow 0 \longrightarrow 90$

where $\phi(\omega) = \tan^{-1}\left(\frac{\omega}{c}\right)$



ω	0		∞
$\phi(\omega)$	0		+90

For $GH(S) = \dfrac{1}{\left(1+\frac{S}{c}\right)}$

نفس اللي فات مع الميل (سالب) وسالب الزاوية

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* $GH(S) = \left(1+\frac{S}{c}\right)^{\pm n}$

C = Corner freq.

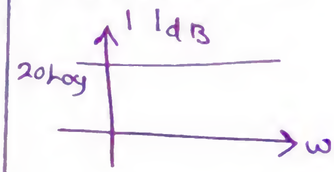
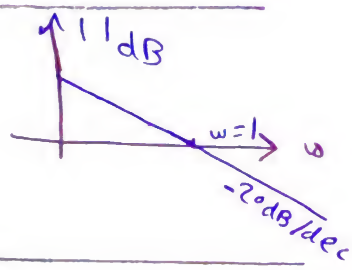
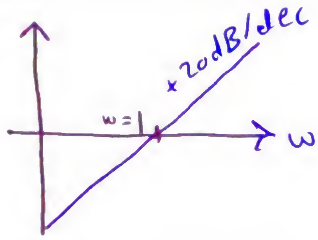
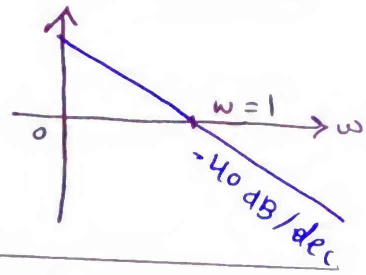
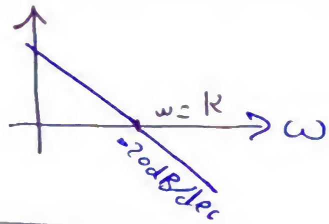
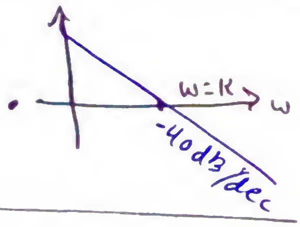
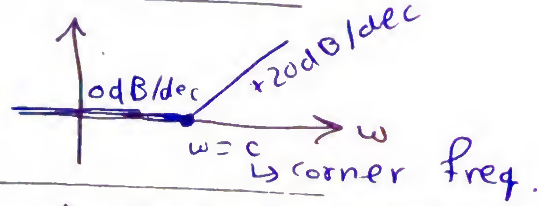
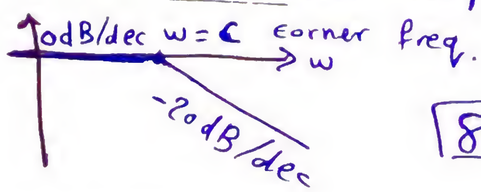
الميل قبل $\omega = c$ = 0 dB/dec

وبعد $\omega = c$ ⇐ الميل = $\pm 20n$ dB/dec

$$\phi(\omega) = \pm n \tan^{-1}(\omega/c)$$

الزاوية محصورة بين $\begin{cases} \omega = 0 \Rightarrow 0 \\ \omega = \infty \Rightarrow \pm n * 90 \end{cases}$

(ملخص)

Term	$\varphi(\omega)$	$\vert\ \vert_{dB}$
K	0	
$\frac{1}{S} \rightarrow \frac{1}{j\omega}$	-90	
$S \rightarrow j\omega$	$+90$	
$\frac{1}{S^2} \Rightarrow \frac{1}{j\omega . j\omega}$	-180	
$\frac{K}{S} \rightarrow \frac{K}{j\omega}$	-90	
$\frac{K}{S^2} \rightarrow \frac{K}{j\omega , j\omega}$	-180	
$1+\frac{S}{c}$ $\Rightarrow 1+j\frac{\omega}{c}$	$\tan^{-1}\left(\frac{\omega}{c}\right)$	
$1/(1+\frac{S}{c})$ $\Rightarrow \frac{1}{(1+j\frac{\omega}{c})}$	$\tan^{-1}\left(\frac{\omega}{c}\right)$	

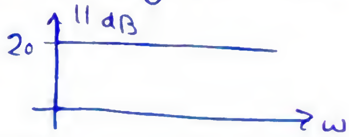
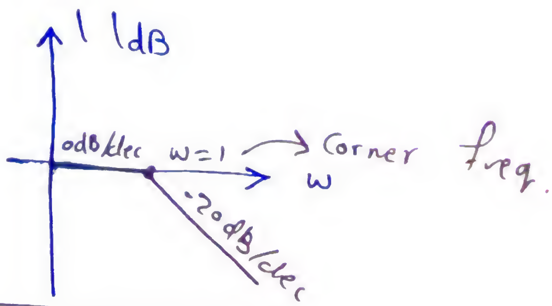
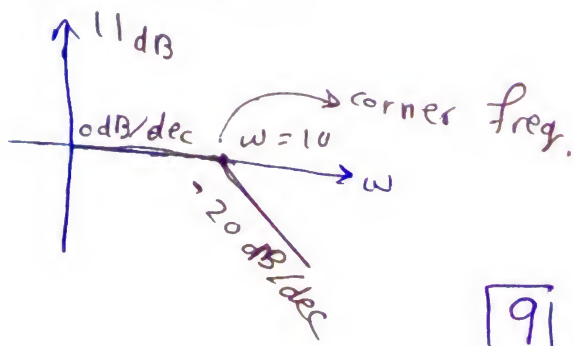
Ex: $$GH(s) = \frac{10}{(1+\underset{T}{s})(1+\underset{\frac{1}{10}}{0.1}s)}$$

Draw the Bode Diagram and find GM & PM

$s \rightarrow j\omega$

$$GH(j\omega) = \frac{10}{(1+j\omega)(1+0.1j\omega)} = \frac{10}{(1+j\frac{\omega}{1})(1+j\frac{\omega}{10})}$$

$$\phi(\omega) = \underset{\text{بسط}}{0} - \underset{\frac{1}{(1+j\omega)}}{\tan^{-1}(\omega)} - \underset{\frac{1}{1+j\frac{\omega}{10}}}{\tan^{-1}\left(\frac{\omega}{10}\right)}$$

term	$\phi(\omega)$	$\lvert\ \rvert_{dB}$
10	0	$20 \log 10 = 20$ 
$\frac{1}{1+\frac{s}{1}} \Rightarrow \frac{1}{1+j\frac{\omega}{1}}$	$-\tan^{-1}(\omega)$	
$\frac{1}{1+\frac{s}{10}} \Rightarrow \frac{1}{1+j\frac{\omega}{10}}$	$-\tan\left(\frac{\omega}{10}\right)$	

ω_{gc}

ω	0	0.1	1	10	100	∞
$\phi(\omega)$	0	-6.3	-50.7	-129.3	-173.7°	-180

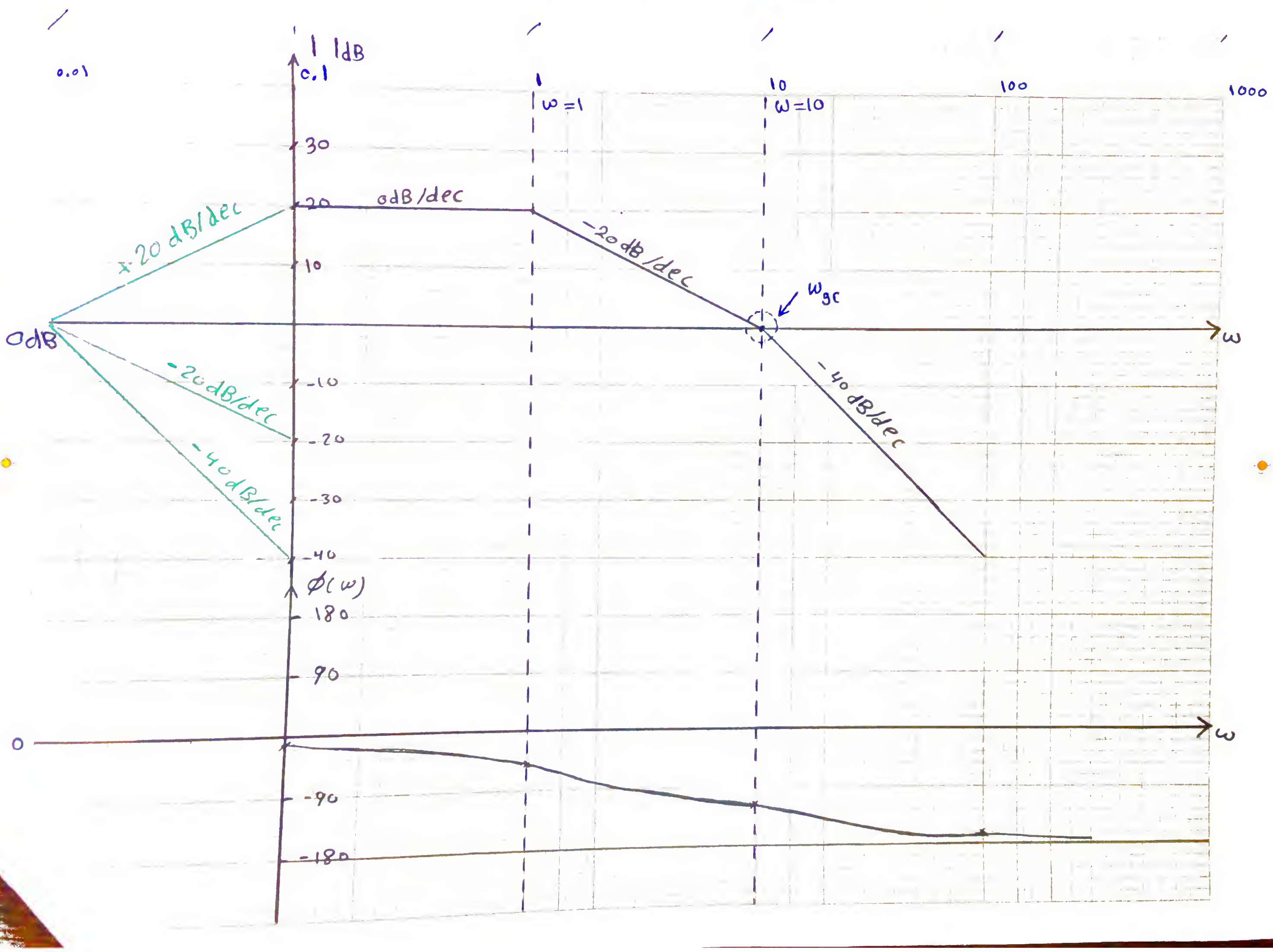
phase ضروري لرسم ال

$$PM = 180 + \phi(\omega = \omega_{gc})$$

$$= 180 - 129.3 = + \checkmark$$

$$GM = \infty$$

$\Rightarrow$ Stable System



0.01
0.1
1
10
100
1000
| |dB
ω=1
ω=10
30
20
10
0dB/dec
+20 dB/dec
-20 dB/dec
ω_gc
0dB
ω
-20dB/dec
-40dB/dec
-10
-20
-30
-40
ϕ(ω)
180
90
0
-90
-180